numbers increase in the exponential, the series in (4.1) converges no more slowly than the sum of the terms of an infinitely decreasing geometric progression, while the solution of the integral Eqs.(4.2) can be obtained by using a large set of effective methods including the asymptotic methods developed for a similar class of equations (/10, 11/, for instance).

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DYNAMIC PROPERTIES OF AN ELASTIC SEMIBOUNDED MEDIUM IN THE PRESENCE OF TWO MASSIVE STAMPS*

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The dynamic properties of a system consisting of two massive rigid strip stamps and an elastic semi-infinite medium are investigated. A layer, a cylinder, a multilayer foundation, etc., can be selected as such a medium. The method of fictitious absorption is used, which are developed for one stamp in /1/. Unlike other approaches to solve there problems /2-4/, this method enables one to describe, to any degree of accuracy, the behaviour of contact stresses simultaneously at all points of the contact domain, both inside and on the boundary.

The presence of resonance frequencies of four kinds is established in the system. Among the first kind is the value of the frequency $\varkappa_{2*}$, starting with which the system has no energetic solution and waves propagate therein that have only geometric damping. The critical frequency here is independent of the stamp characteristics and is determined just by the geometric and dynamic properties of the waveguide. The second kind of resonances is characterized by the frequencies to which multiple roots correspond, i.e., the poles of the

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integrand $K(\lambda)$ of the kernel of the system of integral equations if the wavelength is $2\pi/\lambda \neq \infty$. The third kind of resonances is obtained if $K(\lambda)$ has multiple poles for $\lambda = 0$ /2/. In the case of resonances of the third kind both the resonant frequencies and the vibrations amplitudes have finite relative maxima and become infinite only for zero mass. There is always a denumerable set of resonances of the third kind. It is actually asserted in /4/ that the amplitude at these resonances is infinite for any mass not equal to zero.

Resonances of a fourth kind, the B resonances predicted in /5/, are detected for stamps in this paper. The same resonances were found earlier in the problem of one stamp /6/.*(*See also: Vorovich E.I., Pryakhina O.D. and Tukodova O.M., Wave excitation by a massive stamp on an elastic layer. Dep. No. 7641-84, VINITI, Moscow, 1984.) The number of these resonances is always finite, they are located on a segment prior to the first resonance frequency $\varkappa_{2*}$, occur for masses greater than a certain critical value m_*, and the vibration amplitude thereon is infinite for any mass $m \geqslant m_*$. In particular, two resonant frequencies of the B kind are detected for the system examined in this paper.

1. Let two strip stamps of mass m_1 and m_2 be in frictionless contact with an elastic semibounded medium. Harmonic forces $P_1e^{-i\omega t}$ and $P_2e^{-i\omega t}$ are applied to the stamp centre of mass, whereupon the stamps perform periodic vibrations consisting of translational displacement of the centres of mass and rotations around the centres of gravity.

The stamp displacements and the wave field excited in the medium and determined from the joint solution of the equations of motion of the stamps and the medium /2/. The contact condition should be satisfied here: equality of the displacement amplitudes of the stamp sole points and the medium surface points in the contact domain ($a_{2k-1} \leqslant x \leqslant a_{2k}$, $-\infty < y < \infty$, $z = 0$, $k = 1, 2$). The problem mentioned is reduced to the solution of a system of integral equations of the first kind by the method of integral transforms, which system is written in the following form, taking the contact condition into account (in dimensionless amplitude parameters):

$$Gq_1 + Gq_2 = u_{1c} + \varphi_1 (x - x_{1c}),\ x \in (a_1, a_2) \tag{1.1}$$

$$Gq_1 + Gq_2 = u_{2c} + \varphi_2 (x - x_{2c}),\ x \in (a_3, a_4)$$

$$Gq_i = \int_{a_{2i-1}}^{a_{2i}} k(x-\xi) q(\xi)\, d\xi$$

$$k(x) = \frac{1}{2\pi} \int_\sigma K(\alpha) e^{-i\alpha x}\, d\alpha \tag{1.2}$$

Here u_{kc} are vertical displacements of the centre of mass with the coordinate $x_{kc} = (a_{2k} + a_{2k-1})/2$, $\varphi_k{}'$ are the angles of rotation around the horizontal axis passing through the centre of gravity, and q_k are the contact stresses characterizing the reaction of the medium for the first $(k = 1)$, and second $(k = 2)$ stamps, respectively.

The form of the function $K(\alpha)$ is determined by the kind of medium, where $K(\alpha)$ possesses the properties of evenness, is real for a real argument, and allows representation in the form of the ratio of two entire functions with conservation of the behaviour of the form $c|\alpha|^{-1}$ at infinity. The contour σ is selected from the energy radiation condition at infinity, which ensures uniqueness of the solution of the problem /2/.

The displacements u_{kc} and the angles of rotation φ_k of the stamps satisfy the equations of motion of a solid (in dimensionless amplitude parameters)

$$-\varkappa_2^2 m_k u_{kc} = P_k - Q_k,\ \varkappa_2^2 = \rho\omega^2 h^2/\mu \tag{1.3}$$

$$-\varkappa_2^2 J_k \varphi_k = M_k - N_k,\ J_k = m_k (a_{2k} - a_{2k-1})^2/12,\ k = 1,2$$

$$Q_k = \int_{a_{2k-1}}^{a_{2k}} q_k(x)\, dx,\quad N_k = \int_{a_{2k-1}}^{a_{2k}} q_k(x)(x - x_{kc})\, dx \tag{1.4}$$

where ω is the frequency of vibration, ρ is the density of the medium, μ is the Lame parameter, h is the characteristic dimension of the medium (the layer thickness, say), J_k are the stamp moments of inertia with respect to the horizontal axes passing through the centres of mass, and Q_k, N_k are the amplitudes of the equal-acting forces and moments of the contact stresses.

The stamp total displacement will be determined from the formula

$$u_k = u_{kc} + \varphi_k (x - x_{kc}),\ k = 1,2 \tag{1.5}$$

2. Let us consider the system of integral equations

$$Gq_1 + Gq_2 = f_1,\quad x \in (a_1, a_2),\quad Gq_1 + Gq_2 = f_2,\quad x \in (a_3, a_4) \tag{2.1}$$

where the integral operator G has the form (1.2), and $f_{1,2}$ are given functions. The method of fictitious absorption /1/ is used to solve system (2.1).

We represent the integrand of the kernel of the operator (1.2) in the form of the product of two functions $K(\alpha) = S(\alpha)\Pi(\alpha)$, where $S(\alpha)$ has a behaviour at infinity that agrees with

$K(\alpha)$, while $\Pi(\alpha)$ contains all the singularities of $K(\alpha)$ on the real axis and possesses the property $\Pi(\alpha) \to 1$ as $|\alpha| \to \infty$. We can select $c(\alpha^2 + B^2)^{-1/2}$ or $\alpha^{-1} \operatorname{th} B\alpha$ as $S(\alpha)$ where B is an arbitrary parameter with $B \gg 1$ in conformity with the requirements of the method of fictitious absorption.

To construct an approximate solution of the integral equation, we approximate the function $\Pi(\alpha)$ by the expression

$$\Pi_*(\alpha) = \prod_{k=1}^{n} (\alpha^2 - z_k^2)(\alpha^2 - p_k^2)^{-1}$$

to a given degree of accuracy. According to the theorems established in /2/, this ensures the closeness of the solution of the system of integral equations with the kernels $K = S\Pi$ and $K_* = S\Pi_*$.

We seek the solution of system (2.1) by the method of fictitious absorption in the form of the sum

$$q_m(x) = q_m^\circ(x) + \varphi_m(x), \quad m = 1,2 \tag{2.2}$$

so that the equalities

$$\int_{a_{2m-1}}^{a_{2m}} q_m^\circ(x) e^{\pm i p_k x} dx = 0, \quad k = 1, 2, \ldots, n, \quad m = 1, 2$$

are satisfied, where p_k are poles of the function $\Pi(\alpha)$ and such that $\operatorname{Im} p_k \geqslant 0$. It is convenient to take a system of delta functions with non-intersecting supports at the points $x_{km} = a_{2m-1} + k(a_{2m} - a_{2m-1})/(2n+1)$ that divide the intervals (a_{2m-1}, a_{2m}) into the equal segments

$$\varphi_m(x) = \sum_{k=1}^{2n} c_{km} \delta(x - x_{km}) \tag{2.3}$$

as the component $\varphi_m(x)$ (c_{km} are constants to be determined).

We introduce the new unknown functions $t_m(x)$ by the relationships

$$t_m(x) = \frac{1}{2\pi} \int_\sigma T_m(\alpha) e^{-i\alpha x} d\alpha, \quad m = 1, 2 \tag{2.4}$$

$$T_m(\alpha) = \Pi(\alpha) Q_m^\circ(\alpha), \quad Q_m^\circ(\alpha) = \int_{a_{2m-1}}^{a_{2m}} q_m^\circ(x) e^{i\alpha x} dx$$

Inserting (2.2) and (2.3) into (2.1) and taking account of (2.4), we arrive at a system of equations in the new unknowns $t_m(x)$

$$\begin{gathered} Gt_1 + Gt_2 = F_1, \; x \in (a_1, a_2) \\ Gt_1 + Gt_2 = F_2, \; x \in (a_3, a_4) \\ F_m = f_m(x) - \sum_{k=1}^{2n} \sum_{l=1}^{2} c_{kl} k(x - x_{kl}), \quad m = 1, 2 \end{gathered} \tag{2.5}$$

The kernel of the integral operator G defined by (1.2) has the form

$$k(x) = \frac{1}{2\pi} \int_\sigma S(\alpha) e^{-i\alpha x} d\alpha$$

We take $(\alpha^2 + B^2)^{-1/2}$ as $S(\alpha)$. The contour σ agrees with the real axis in this case. Without loss of generality, we also set $f_m(x) = A_m e^{-i\eta x}, \operatorname{Im} \eta = 0, \eta > 0$. Therefore, the system of integral equations of the dynamic contact problem (2.1) has been reduced to a system of equations of the static problem (2.5) with integrand of the kernel $S(\alpha)$ that has no singularities on the real axis.

The solution of system (2.5) can be constructed by numerous methods for solving static problems or problems for media with strong absorption by virtue of the rapid decrease of the integrand of the kernel $S(\alpha)$. For example, we obtain by the method of factorization

$$t_m(x) = A_m t_m^\eta(x) - \frac{1}{2\pi} \sum_{k=1}^{2n} \sum_{l=1}^{2} c_{kl} \int_\sigma L_l(\alpha, x_{kl}) e^{-i\alpha x} d\alpha, \quad m = 1, 2 \tag{2.6}$$

The functions $L_l(\alpha, x_{kl})$ and $t_m^\eta(x)$ have the form

$$L_l(\alpha,\xi)=\frac{1}{2\pi}\int_\sigma S(\eta)\,\Pi(\eta)\,T_l^\eta(\alpha)\,e^{i\eta\xi}\,d\eta \tag{2.7}$$

$$T_l^\eta(\alpha)=\int_{a_{2l-1}}^{a_{2l}} t_l^\eta(x)\,e^{i\alpha x}\,dx,\quad l=1,2$$

$$t_l^\eta(x)=u_l^\eta(x)+\sqrt{B^2+\eta^2}\,e^{-i\eta x}\,[v_l^\eta(x)-1]$$

$$u_l^\eta(x)=\frac{\sqrt{B-i\eta}\exp(-i\eta a_{2l})}{\sqrt{\pi(a_{2l}-x)}}\exp[-B(a_{2l}-x)]+$$
$$\frac{\sqrt{B+i\eta}\exp(-i\eta a_{2l-1})}{\sqrt{\pi(x-a_{2l-1})}}\exp[-B(x-a_{2l-1})]$$

$$v_l^\eta(x)=\operatorname{erf}\sqrt{(B+i\eta)(a_{2l}-x)}+\operatorname{erf}\sqrt{(B-i\eta)(x-a_{2l-1})}$$

On the basis of the lemma (/7/, p.168), for the functions $q_m^\circ(x)$ to have a support in the domain Ω: $a_{2m-1}\leqslant x\leqslant a_{2m}$, it is necessary and sufficient that

$$T_m(\pm z_k)=0,\ k=1,2,\ldots,n \tag{2.8}$$

(z_k are the zeros of the function $\Pi(\alpha)$, such that $\operatorname{Im} z_k\geqslant 0$).

The relationships (2.8) are $4n$ equalities in $4n$ unknown functions c_{km} ($k=1,2,\ldots,2n$, $m=1,2$). By the theory of residues these relationships can be reduced to the following linear algebraic system to determine the coefficients c_{km}

$$\sum_{k=1}^{2n}\sum_{l=1}^{2} c_{kl}\,[\sqrt{B+i\alpha}\exp(i\alpha a_{2m})F_{l-s}(\alpha,a_{2m}-x_{kl})+ \tag{2.9}$$
$$\sqrt{B-i\alpha}\exp(i\alpha a_{2m-1})F_{m-s}(-\alpha,x_{kl}-a_{2m-1})]=$$
$$A_mT_m^\eta(\alpha),\quad m=1,2,\quad \alpha=\pm z_i,\quad i=1,2,\ldots,n$$

($s=1$ for $l=m=2$, in the remaining cases $s=0$).

Here

$$F_l(\alpha,\xi)=\sum_{j=1}^{n}\alpha_j\frac{e^{\pm ip_j\xi}}{2p_j(\alpha\pm p_j)\sqrt{B+ip_j}},\quad l=1,2$$

($l=1$ corresponds to the plus sign, and $l=2$ to the minus sign).

Having determined c_{km} from system (2.9), we find the functions $q_m^\circ(x)$ and, therefore, also $q_m(x)$ by means of (2.4) and (2.2), where

$$q_m^\circ(x)=\frac{1}{2\pi}\int_\sigma T_m(\alpha)\,\Pi^{-1}(\alpha)\,e^{-i\alpha x}\,d\alpha \tag{2.10}$$

$$T_m(\alpha)=\int_{a_{2m-1}}^{a_{2m}} t_m(x)\,e^{i\alpha x}\,d\alpha$$

We obtain the final formulas to compute the contact stresses $q_m(x)$ under the stamps. We insert expression (2.6) for $t_m(x)$ into the integral representation of the solution (2.10) and we use the approximation $\Pi(\alpha)$. The integrals L_m $(m=1,2)$, defined by (2.7) are taken in residues since the integrands decrease exponentially in the lower half-plane of the complex variable η and have no branch points there. The remaining integrals in the solution (2.10) are calculated by means of the formulas of the operational calculus.

Omitting the calculations, we present the general form of the approximate solution of system (2.1)

$$q_m(x,\eta)=\frac{A_m}{c}\Big\{u_m^\eta(x)-K^{-1}(\eta)e^{-i\eta x}+K^{-1}(\eta)e^{-i\eta x}v_m^\eta(x)+ \tag{2.11}$$
$$\sum_{r=1}^{n}\frac{\beta_r}{2z_r}\,[\exp(-i\eta a_{2m})\sqrt{B-i\eta}\,\Phi_r(\eta,a_{2m}-x)+$$
$$\exp(-i\eta a_{2m-1})\sqrt{B-i\eta}\,\Phi_r(-\eta,x-a_{2m-1})]\Big\}-$$
$$\frac{i}{c}\sum_{l=1}^{2}\sum_{k=1}^{2n}c_{kl}\Big\{\frac{\exp[-B(a_{2m}-x)]}{\sqrt{\pi(a_{2m}-x)}}P_{l-s}(a_{2m}-x_{kl})+$$
$$\frac{\exp[-B(x-a_{2m-1})]}{\sqrt{\pi(x-a_{2m-1})}}P_{m-s}(x_{kl}-a_{2m-1})+$$
$$\Psi_{l-s}(a_{2m}-x_{kl},a_{2m}-x)+\Psi_{m-s}(x_{kl}-a_{2m-1},x-a_{2m-1})\Big\}$$
$$a_{2m-1}\leqslant x\leqslant a_{2m},\ m=1,2$$

The parameter is $s=1$ for $l=m=2$ and $s=0$ in the remaining cases.
The following notation was used

$$P_l=\sum_{j=1}^{n}\frac{\alpha_j\exp(\pm ip_jx)}{2p_j\sqrt{B\mp ip_j}}$$

$$\Psi_l(x,y)=\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\beta_i\alpha_j}{4p_jz_i}\,\frac{\exp(\pm ip_jx)}{2p_j\sqrt{B+ip_j}}\,\Phi_i(\mp p_j,y)$$

$$\Phi_r(x,y)=\frac{\sqrt{B+iz_r}}{z_r-x}\exp(iz_ry)\,\mathrm{erf}\sqrt{(B+iz_r)y}+\frac{\sqrt{B-iz_r}}{-z_r-x}\exp(-iz_ry)\left[1-\mathrm{erf}\sqrt{(B-iz_r)y}\right]$$

$$\alpha_j=\prod_{k=1}^{n}(p_j^2-z_k^2)\prod_{\substack{k=1\\ j\neq k}}^{n}(p_j^2-p_k^2)^{-1}$$

$$\beta_i=\prod_{k=1}^{n}(z_i^2-p_k^2)\prod_{\substack{k=1\\ i\neq k}}^{n}(z_i^2-z_k^2)^{-1}$$

A program package has been compiled for the BESM-6 to compute the contact stresses $q_m(x)$ and the coefficients c_{km} occurring in the solution.

As an illustration, we consider a layer adhering rigidly to a non-deformable foundation. The form of the function $K(\alpha)$ is given in /1, 2/ and is not presented here. Graphs of the real part of the contact stress amplitudes $\mathrm{Re}\,q_1$ and $\mathrm{Re}\,q_2$ are represented in Fig.1 for given unit displacements of the stamps of identical width $(A_1=A_2=1)$ for different distances between the stamps: $2b=2,6,10$ (curves *1-3*).

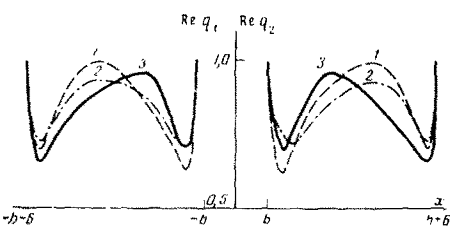

Fig.1

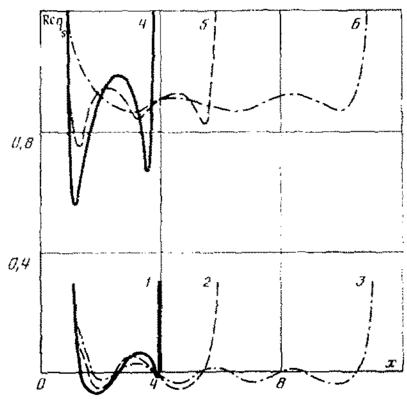

Fig.2

Fig.2 characterizes the contact pressures $\mathrm{Re}\,q_1$ that occur under a stamp at rest for a given unit motion of the other $(A_1=0,\ A_2=1)$ for different values of the stamp width $a_2-a_1=6,10,20$ (lines *1-3*). Lines *4-6* describe the behaviour of $\mathrm{Re}\,q_1$ when both stamps vibrate translationally with unit amplitude $(A_1=A_2=1)$. All the graphs are given for $\varkappa_2=2{,}6;\ \eta=0$ and $a_4-a_3=6$. Analysis shows that the stress oscillation increases with the stamp dimension while the amplitude diminishes.

We note that the real loads referred to μh^2 are quantities of the order of $10^{-8}-10^{-12}$. Consequently, when going over to specific parameters the results presented in the graphs and obtained for unit displacement decrease by many orders.

3. Let $q_{1,2}^j\ (j=1,2,3,4)$ be solutions of the system of integral equations (2.1) for the right-hand sides

$$\begin{pmatrix}1\\0\end{pmatrix},\quad\begin{pmatrix}0\\1\end{pmatrix},\quad\begin{pmatrix}x\\0\end{pmatrix},\quad\begin{pmatrix}0\\x\end{pmatrix}$$

respectively. Then by virtue of the linearity of the problem, the solution of the system (1.1) will be

$$q_k=(u_{1c}-\varphi_1x_{1c})q_k^1+(u_{2c}-\varphi_2x_{2c})q_k^2+\varphi_1q_k^3+\varphi_2q_k^4,\quad k=1,2 \tag{3.1}$$

where $q_k^j(x)$ are related to the solution $q_k(x,\eta)$ obtained in Sect.2 as follows

$$q_k^j(x) = q_k(x, 0), \; A_1 = 1, \; A_2 = 0 \text{ for } j = 1$$

and $A_1 = 0, \; A_2 = 1$ for $j = 2$

$$q_k^j(x) = i\frac{dq_k(x,\eta)}{d\eta}, \quad A_1 = 1, \quad A_2 = 0, \quad \eta = 0 \quad \text{for } j = 3$$

and $A_1 = 0, A_2 = 1, \eta = 0$ for $j = 4$

Substituting (3.1) into (1.3) and (1.4), we obtain a system of four equations to determine the four unknowns $u_{1c}, u_{2c}, \varphi_1$ and φ_2 in the form

$$(\mathbf{A} + \mathbf{B})\mathbf{u} = \mathbf{P} \tag{3.2}$$

$$\mathbf{u} = \{u_{1c}, u_{2c}, \varphi_1, \varphi_2\}, \; \mathbf{P} = \{P_1, P_2, M_1, M_2\}$$

$$a_{kk} = -m_k \varkappa_2^2, \; a_{k+2,k+2} = -J_k \varkappa_2^2, \; k = 1, 2$$

The remaining elements of the matrix $\mathbf{A}$ equal zero. The elements of the matrix $\mathbf{B}$ have the form

$$b_{k1} = Q_k^1, \quad b_{k2} = Q_k^2, \quad b_{k3} = -x_{1c}Q_k^1 + Q_k^3,$$
$$b_{k4} = -x_{2c}Q_k^2 + Q_k^4$$
$$b_{k+2,1} = -x_{kc}Q_k^1 + Q_k^5, \quad b_{k+2,2} = -x_{kc}Q_k^2 + Q_k^6$$
$$b_{k+2,3} = x_{1c}x_{kc}Q_k^1 - x_{1c}Q_k^5 - x_{kc}Q_k^3 + Q_k^7$$
$$b_{k+2,4} = x_{2c}x_{kc}Q_k^2 - x_{2c}Q_k^6 - x_{kc}Q_k^4 + Q_k^8, \quad k = 1, 2$$
$$\left(Q_k^j = \int_{a_{2k-1}}^{a_{2k}} q_k^j(x)\,dx, \quad Q_k^{j+4} = \int_{a_{2k-1}}^{a_{2k}} q_k^j(x)\,x\,dx, \quad j = 1, 2, 3, 4\right)$$

The general (total) vertical displacements of points of the base of the stamp will be determined from (1.5).

We note that for frequencies less than the critical wave propagation frequency $\varkappa_{2*}$ in the above-mentioned media, the functions Q_k^j are real, while for $\varkappa_2 > \varkappa_{2*}$ they become complex. The frequency $\varkappa_{2*}$ characterizes resonance of the first kind and for a layer $\varkappa_{2*} = \pi/2$. Consequently, it is interesting to study the frequencies $\varkappa_2 < \varkappa_{2*}$ at which the displacement amplitudes $u_k\,(k = 1, 2)$ can become infinite, i.e., those relationships of the problem parameters for which the determinant of the system (3.2) vanishes.

At layer frequencies corresponding to double poles of the function $K(\alpha)$ equal to zero and located in the domain $\varkappa_2 \geqslant \varkappa_{2*}$, the amplitude of the total forces vanishes (which satisfies the requirements of the theory by virtue of a well-known theorem /2/, p.239), and the vibration amplitudes will be infinite for zero mass of the stamp. These resonances of the third kind were studied in /4/. The vibration amplitude becomes finite if the body mass is different from zero, but resonances of another kind (the *B* kind) can appear for $\varkappa_2 < \varkappa_{2*}$ (predicted in /5/ and established in /6/ for the case of one stamp and in this paper for a system of two stamps) at which the vibration amplitude becomes infinite.

We consider the case of a symmetric stamp arrangement and symmetrical loading by unit forces on a layer rigidly adherent to a non-deformable base. In this case the system of four Eqs.(3.2) reduces to a system of second-order equations with two unknowns u_c and φ ($m_1 = m_2 = m, \; u_{1c} = u_{2c} = u_c, \; \varphi_1 = \varphi_2 = \varphi, \; x_{1c} = x_{2c} = x_c$)

$$u_c(-m\varkappa_2^2 + k_1) + \varphi_1(k_3 + x_ck_1) = 1, \quad \varepsilon = (a_2 - a_1)^2/12$$
$$u_c(x_ck_1 + k_2) + \varphi_1[-\varkappa_2^2m\varepsilon + (x_c)^2k_1 + x_c(k_2 + k_3) + k_4] = 1$$

Equating the determinant of this system to zero, we arrive at the equation

$$m^2\varkappa_2^4 - 2m\varkappa_2^2 f_1(\varkappa_2) = f_2(\varkappa_2) \tag{3.3}$$
$$f_1(\varkappa_2) = [(x_c^2 + \varepsilon)k_1 + x_c(k_2 + k_3) + k_4]/(2\varepsilon)$$
$$f_2(\varkappa_2) = (k_3k_2 - k_1k_4)/\varepsilon$$
$$(k_1 = Q_1^1 + Q_1^2, \quad k_2 = Q_1^5 + Q_1^6, \quad k_3 = Q_1^3 - Q_1^4,$$
$$k_4 = Q_1^7 - Q_1^8)$$

Eq.(3.3) can have two positive real roots if its discriminant *D* is positive, one root if $D = 0$ and none when $D < 0$.

In Fig.3 we show the dependence of the resonance frequencies $\varkappa_2$ on the stamp mass *m* for a given stamp width $a_2 - a_1 = 6$. It is seen that two critical masses exist, m_{1*} and m_{2*} such that when the stamps possess a mass less than m_{1*} (which corresponds to $D < 0$), there is no resonance in the system. For $m_{1*} < m < m_{2*}$ there is one resonance. And finally, for sufficiently large masses $m \geqslant m_{2*}$ the system has two resonance frequencies at which the amplitude of the forced steady vibrations becomes infinite. As the mass grows the value of the resonance frequencies falls. The behaviour of the resonance curves does not change qualitatively as the stamp dimensions change.

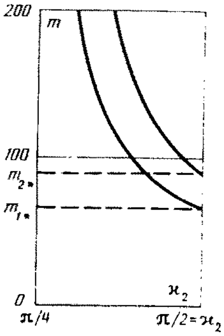

Fig.3

Therefore, the system under consideration cannot have more than two resonances in the domain $\varkappa_2 \leqslant \varkappa_{2*}$. Therefore, resonances of the kind *B* exist when $\det(\mathbf{A}+\mathbf{B})=0$ and such resonances are obviously possible only for frequencies less than the frequency $\varkappa_{2*}$ corresponding to the initial point of a continuous spectrum. For semi-infinite bodies these resonances characterize the fact that the vibration amplitudes become infinite, which compares a semi-infinite body with a body of bounded dimensions in its dynamic properties.

For resonances at $\varkappa_2 > \varkappa_{2*}$ the vibration amplitude takes a maximum but quite definite finite value. Precisely these resonances (of the third kind) were investigated earlier /4/. We note that the determinant for $\varkappa_2 > \varkappa_{2*}$ generally never vanishes for a non-zero stamp mass although this was asserted in /4/. Moreover, resonances of the *B* kind in a layer or strip with stamps only occur for a fairly large stamp mass, starting with a certain critical value while resonances of the third kind occur for any stamp mass.

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